# ON V-SYMMETRY IN SU (3)

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ABSTRACT. Some properties of $V$-symmetry in SU(3) have been discussed here.

## INTRODUCTION

$V$-symmetry is one of the subgroups of $SU(3)$. It appears that in different pieces of literature (Dalitz, 1963; Mayer, 1963; Low, 1964; Sakurai, 1964; Lipkin, 1966; Matthews, 1966; Swart, 1966), the other two subgroups, $I$-symmetry and $U$-symmetry, have been studied in comparatively greater detail. We propose to consider certain aspects of $V$-symmetry in this paper.

Now we briefly introduce $V$-symmetry in a general way. Let us consider three unit vectors in $F_3-F_8$ or $I_3-Y$ plane [$F_i$ being the infinitesimal generators in the basic representation of $SU(3)$] :

$$\vec{i} = (1,0), \quad \vec{u} = (-\tfrac{1}{2}, \sqrt{3}/2), \quad \vec{v} = (\tfrac{1}{2}, \sqrt{3}/2) \qquad \text{.. (1)}$$

and define the vector generator :

$$\vec{E} = (F_3, F_8) \qquad \text{.. (2)}$$

and put

$$\vec{i}.\vec{E} = I_3, \quad \vec{u}.\vec{E} = U_3, \quad \vec{v}.\vec{E} = V_3 \qquad \text{.. (3)}$$

Then,

$$I_3 + U_3 - V_3 = 0 \qquad \text{.. (4)}$$

Also we define :

$$\begin{aligned} I_\pm &= F_1 \pm iF_2 \\ U_\pm &= F_6 \pm iF_7 \\ V_\pm &= F_4 \pm iF_5 \end{aligned} \qquad \text{.. (5)}$$

Thus we obtain the three subgroups $(I\ ,\ I_3)$, $(U\ ,\ U_3)$ and $(V\ ,\ V_3)$, of which the first is just iso-spin ($I$-spin) and the others are $U$-spin and $V$-spin respectively. $(I_1, I_2)$, $(U_1, U_2)$ and $(V_1, V_2)$ are given by :

$$I_1 = \frac{I_+ + I_-}{2} \quad I_2 \frac{I_+ - I_-}{2} \quad U_1 = \frac{U_+ + U_-}{2}, \quad U_2 = \frac{U_+ - U_-}{2}$$

$$V_1 \quad \frac{V_+ + V_-}{2}, \quad V_2 \quad \frac{V_+ - V_-}{2} \qquad (6)$$

PARTICLES AT ORIGIN OF WEIGHT DIAGRAMS

In a weight diagram (figure 1), just as the horizontal lines parallel to $\vec{i}$ link particles of the same $I$-sping multipletes, so the lines parallel to $\vec{v}$ connect $V$-multiplets. Examples of $V$—spin multiplets are :

$(K^\circ, \pi^-)$, $(\pi^+, \bar{K}^0)$, $(n, \Sigma^-)$, $(\Sigma^+, \Xi^0)$—(fig. 2 and 5)

$(\Delta^{++}, Y_1^{*+}, \Xi^{*\circ}, \Omega^-)(Y^{**+}, \Xi^{**0})$—(fig. 3 and 4).

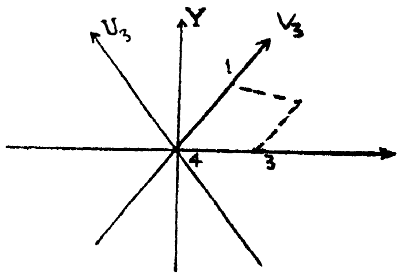


Figure 1,

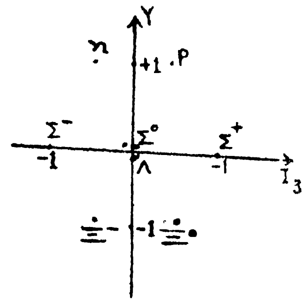


Figure 2.

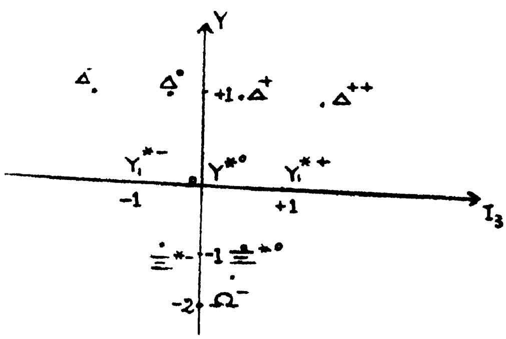


Figure 3.

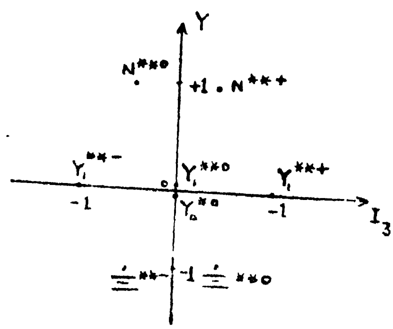


Figure 4,

Where there are two particles at the origin on a weight diagram (say, figure 5), we may define in the $F_3$—$F_8$ plane a vector (Matthews)

$$\vec{\pi^0} = (\pi^0, \eta)$$

Then, the $I_3 = 0$ component of the I-spin triplet is

$$\pi^0{}_i = \vec{\pi^0}.\vec{i} = \pi^0,$$

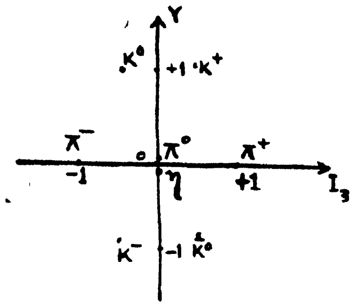


Fig. 5.

and the $I$-spin singlet orthogonal to it is

$$\eta_t = \eta$$

Similarly, the $V_3 = 0$ component of the $V$-spin triplet (figure 5) is from (1) :

$$\pi^{\circ}_{v} = \vec{\pi^0}.\ \vec{v} = \frac{\pi^0 + \sqrt{3}}{2} \qquad (7)$$

and

$$\eta_v = \frac{\sqrt{3}\pi^0 + \eta}{2}$$

Also, for the baryon octet (figure 2)

$$\Sigma_v^{\ 0} = \frac{\Sigma^0 + \sqrt{3}\Lambda}{2}$$

$$\Lambda_v = \frac{\sqrt{3}\Sigma^0 + \Lambda}{2} \qquad (8)$$

and for the baryon resonance (figure 4)

$$Y_{1v}^{\ **0} = \frac{Y*_1{}^{*0} + \sqrt{3}Y}{2}$$

$$Y_{0v}^{\ *0} = \frac{\sqrt{3}Y_1^{\ **0} + Y_0^{\ *0}}{2} \qquad (9)$$

### $V_2$-SYMMETRY

Following the cases of $I$-spin and $U$-spin, let us consider the discrete transformation :

$$P_v = \exp[i\ V_2\pi] \qquad (10)$$

This will have the effect of reflecting the weight diagrams in the $V_3 = 0$ axis. Then, we should be should have the following transformations :

$$K^0 \longleftrightarrow \pi^-, \;\; K^+ \longleftrightarrow K^-, \;\; \pi^+ \longleftrightarrow \bar{K}^0, \;\; n \longleftrightarrow \Sigma^-, \;\; p \longleftrightarrow \Xi^- \;\; \Sigma^+ \longleftrightarrow \Xi^0,$$

$$\Delta^0 \longleftrightarrow Y_1^{*-}, \;\; \Delta^+ \longleftrightarrow \Xi^-, \;\; \Delta^{++} \longleftrightarrow \Omega^-$$

Invariance under these transformations implies that

$$<f|S|i> = <f|P_v^+ S P_v|i> \qquad .. \quad (11)$$

where $<f|S|i>$ stands for the $S$-matrx between the initial state $|i>$ and the final state $<f|$. Then we should expect :

$$<\pi^-\Sigma^+|S|\Sigma^-\pi^+> = <K^0\,\Xi^0|S|n\bar{K}^0>$$

$$<K^-p|S|\Xi^-K^+ = <K^+\,\Xi^-|S|pK^-> \qquad .. \quad (12)$$

Again, since $\Lambda_v \longleftrightarrow \Lambda_v$ under $P_v$-transformation, we may take $\Lambda \longleftrightarrow \Lambda$ from (8). This should give :

$$<\Lambda K^0|S|p\pi^-> = <\Lambda\pi^-|S|\Xi^-K^0>$$

$$<\Lambda K^+|S|n\pi^+> = <\Lambda K^{-1}|S|\Sigma^-\bar{K}^0> \qquad .. \quad (13)$$

## MAGNETIC MOMENTS

For any $V$-multiplet, the charge is given by :

$$Q = V_3 + a \qquad .. \quad (14)$$

Thus, for the doublet $(\Sigma^+, \Xi^0)$, $a = \frac{1}{2}$. For the triplet $(p, \Sigma_v{}^\circ, \Xi^-)$, $a = 0$. For the quadruplet $(\Delta^{++}, Y_1{}^{*+}, \Xi^{*0}, \Omega^-)$ also, $a = \frac{1}{2}$.

Now, since $Q$ depends linearly upon $V_3$, any physical property dependent linearly on electromagnetic interaction may be expressed in the form :

$$\alpha + \beta V_3 \qquad .. \quad (15)$$

for a $V$-spin multiplet. The static magnetic moments for the triplet $(p, \Sigma_v{}^\circ, \Xi^-)$ will be given by

$$\mu(p) - \mu(\Sigma_v{}^\circ) = \mu(\Sigma_v{}^\circ) - \mu(\Xi^-) \qquad .. \quad (16)$$

Now, taking (Mayer)

$$\mu(\Xi^-) = -\mu(p) - \mu(n)$$

we have form (16)

$$\mu(\Sigma_v{}^0) = -\tfrac{1}{2}\mu(n) \qquad .. \quad (16a)$$

On the other hand (Matthews)

$$\mu(\Sigma_u{}^0) = \mu(n) = -2\mu(\Sigma_v{}^0)$$

The full expression for $\mu(\Sigma_v{}^0)$ may be written thus

$$\mu(\Sigma_v{}^0) = \langle \Sigma_v{}^0 \,|\, J \,|\, \Sigma_v{}^0 \rangle = \left\langle \frac{\Sigma^0+\sqrt{3}\Lambda}{2} \right| J \left| \frac{\Sigma^0+\sqrt{3}\Lambda}{2} \right\rangle$$

$$= \frac{1}{4}\mu(\Sigma^0) + \frac{3}{4}\mu(\Lambda) + \frac{\sqrt{3}}{2}\mu(\Lambda\Sigma^0) \qquad (17)$$

Now, it can be shown (Matthews) that

$$\frac{\sqrt{3}}{2}\mu(\Lambda\Sigma^0) = \frac{3}{4}\mu(\Sigma^0) - \frac{3}{4}\mu(\Lambda) \qquad .. \quad (18)$$

combining (17) and (18),

$$\mu(\Sigma_v{}^0) = \mu(\Sigma^0) \qquad .. \quad (19)$$

Also, for the triplet $(\Delta^+,\ Y_1{}^{*0},\ \Xi^{*-})$

$$\mu(\Delta^+) - \mu(Y_1{}^{*0})$$

$$= \mu(Y_1{}^{*0}) - \mu(\Xi^{*-}) \qquad \ldots \quad (20)$$

and for the quadruplet $(\Delta^{++},\ Y_1{}^{*+},\ \Xi^{*0},\ \Omega^-)$

$$\mu(\Delta^{++}) - \mu(Y_1{}^{*+}) = \mu(Y_1{}^{*+}) - \mu(\Xi^{*0}) = \mu(\Xi^{*0}) - \mu(\Omega^-) \qquad \ldots \quad (21)$$

We propose to obtain with $V$-symmetry some well-known mass-relations which have already been derived otherwise.

(a) *Parallelogram and hexagon laws*

## MASS RELATIONS

First, we consider parallelogram law with reference to I-spin and $V$-spin. If there is only one particle at each point in the weight diagram (fig.-1), the law gives :

$$m(1) - m(2) + m(3) - m(4) = 0 \qquad \ldots \quad (22)$$

This follows $I$-spin conservation

$$m(1) = m(2), \quad m(3) = m(4) \qquad \ldots \quad (23)$$

and $V$-spin conservation

$$m(1) = m(4), \quad m(2) = m(3) \tag{24}$$

For the baryon decuplet, we have according to (22)

$$\begin{aligned} &\Delta^{+}-\Delta^{++}+Y_1^{*+}-Y_1^{*o} = 0 \\ &Y_1^{*o}-Y_1^{*+}+\Xi^{*o}-\Xi^{*-} = 0 \\ &\Delta^{0}-\Delta^{+}+Y_1^{*o}-Y_1^{*-} = 0 \end{aligned} \qquad \ldots \quad (25)$$

If, however, there are two particles, say at the point, they may be denoted in $I$-space by and (5) and in $V$-space by and $(5_v)$. Now, we have the relations :

$$m(1)-m(2)+m-m(4)+\alpha m(5)$$

$$-m(1)-m(2)+\beta m(3_v)-m(4)+\gamma m(3_v\, 5_v) = 0 \tag{26}$$

where $m(35)$ and $m(3_v\, 5_v)$ are transition masses in the $I$ and $V$ representations respectively. The constants may be found from the identity (26). However, for $V$-spin conservation only, $\beta$ is assumed to be $= 1$ to satisfy (24).

For the baryon octet, there are two parallelograms. Let us consider the parallelogram $(n, p, \Sigma^0, \Lambda, \Sigma^-)$.
From (26)

$$\begin{aligned} &n-p+\Sigma^0-\Sigma^-+\alpha(\Sigma^0\Lambda) \\ &n-p+\beta\Sigma_v{}^0-\Sigma^-+\gamma(\Sigma_v{}^0\Lambda_v) = 0 \end{aligned} \qquad \ldots \quad (27)$$

It is found from (8) and (27) that (Appendix)

$$\alpha = \sqrt{3}, \quad \beta = -2 \text{ and } \quad \gamma = 2\sqrt{3} \qquad \ldots \quad (23)$$

[It may be stated in passing that for the $I-U$ parallelogram $(n, p, \Sigma^+, \Sigma^0)$, the values are found to be (Appendix-) $\alpha = \sqrt{3}$, $\beta = 1, \gamma = \sqrt{3}$.]
Thus,

$$n-p+\Sigma^0-\Sigma^-+\sqrt{3}(\Sigma^0\Lambda) = 0 \qquad \ldots \quad (29)$$

Similarly, for the other parallelogram.

$$\Sigma^0-\Sigma^++\Xi^0-\Xi^-+\sqrt{3}(\Sigma^0\Lambda) = 0 \tag{30}$$

(29) and (30) quite satisfactorily agree with the corresponding relations obtained with $I-U$ parallelogram (Matthews). From (29) and (30), we obtain the well-known six-mass relation :

$$n-p+\Sigma^+-\Sigma^-+\Xi^--\Xi^0 = 0 \qquad \ldots \quad (31)$$

This relation may also be directly obtained by an extension of parallelogram to hexagon considering conservation of $I$-spin, $U$-sping and $V$-spin. Thus we have (figures 2, 4, 5) :

$$m(1)-m(2)+m(3)-m(4)+m(5)-m(6) = 0$$

This applies to baryon, meson and baryon resonance octets. It thus appears that the siz-mass relation is a characteristic of an octet.

(b) *Electromagnetic and medium-strong interaction effects*

Like $I$-symmetry, $V$-symmetry is broken by electromagnetic interactions, whereas like $U$-symmetry, it is violated by medium-strong interactions. In analogy with either case, it is plausible to write (Matthews) the mass formula as—

$$m = \alpha+\beta V_3+\gamma V_3^2 \tag{32}$$

Then, for the quadruplet ($\Delta^{++}$, $Y_1^{*+}$, $\Xi^{*0}$, $\Omega^-$)

$$\Delta^{++}-2Y_1^{*+}+\Xi^0 = Y_1^{*+}-2\Xi^{*0}+\Omega^- \tag{33}$$

Now,

$$\left.\begin{aligned}\text{L.H.S.} &= 1236-2\times 1382.7+1529.7 = +0.3 \text{ Mev}\\ \text{R.H.S.} &= 1382.7-2\times 1529.7+1675 = -1.7 \text{ Mev}\end{aligned}\right\} \text{(Swart)}$$

The discrepancy between the two sides is to the extent of 2Mev, i.e., about 0.13% only of the average mass.

Following Okubo (Matthews), if it is assumed in this case also that the first-order term dominates in (32), then we have

$$m = \alpha+\beta V_3 \quad .. \tag{34}$$

Applying (34) to the above quadruplet, we obtain exactly the equal spacing rule (Swart) :

$$\Delta^{++}-Y_1^{*+} = Y_1^{*+}-\Xi^{*0} = \Xi^{*0}-\Omega^- \quad .. \tag{35}$$

That is,

$$\left.\begin{aligned}\Delta^{++}-Y_1^{*+} &= 1236-1382.7 = -146.7\\ Y_1^{*+}-\Xi^{*0} &= 1382.7-1529.7 = -147.0\\ \Xi^{*0}-\Omega^- &= 1529.7-1675 = -145.3\end{aligned}\right\} \text{(Swart)}$$

We consider the baryon triplet ($p$, $\Sigma_v^0$, $\Xi^-$) in the light of (34) and obtain

$$p-\Sigma_v^0 = \Sigma_v^0-\Xi^- \quad .. \tag{36}$$

Now,

$$\Sigma_v{}^0 \quad \frac{1}{4}\,\Sigma^0+\frac{3}{4}\,\Lambda+\frac{\sqrt{3}}{2}\,(\Sigma^0\Lambda) \qquad .. \quad (37)$$

(from Appendix)

From (36) and (37)

$$\tfrac{1}{2}(p+\Xi^-) = \Sigma_v = \tfrac{1}{4}\Sigma^0+\tfrac{3}{4}\Lambda+\frac{\sqrt{3}}{2}\,(\Sigma^0\Lambda) \qquad .. \quad (38)$$

Since the average value of the transition mass $(\Sigma^0\Lambda)$ is about 1.15 Mev from (29) and (30), it may be neglected compared to other masses and we obtain

$$\tfrac{1}{2}(p+\Xi^-) = \tfrac{1}{4}\,\Sigma^0+\tfrac{3}{4}\,\Lambda \qquad .. \quad (39)$$

This is just a form of the well-known baryon mass-relation (Swart, Mayer) :

$$\tfrac{1}{2}(M_N+M_\Xi) = \tfrac{1}{4}(M_\Sigma+3M_\Lambda) \qquad .. \quad (40)$$

Applying (34) to the meson triplet $(K^+, \pi_v{}^0, K^-)$ we have as above (masses squared)

$$\tfrac{1}{2}(K^++K^-) = \pi_v{}^0 = \tfrac{1}{4}\,\pi^0+\tfrac{3}{4}\eta+\frac{\sqrt{3}}{2}\,(\pi^0\eta) \qquad .. \quad (41)$$

Neglecting the transition mass $(\pi^0\eta)$ as before,

$$\tfrac{1}{2}(K^++K^-) = \tfrac{1}{4}\pi^0+\tfrac{3}{4}\,\eta \qquad .. \quad (42)$$

This is also a form of the well-known meson mass-relation (Swart, Mayer) :

$$m_k{}^2 = \tfrac{1}{4}(m_\pi{}^2+3m_\eta{}^2) \qquad .. \quad (42)$$

It is interesting that a number of mass-relations may be obtained from simple premises on $V$-symmetry. The above results also vindicate Okubo's suggestion which has been extended to $V$-spin here.

## ELECTROMAGNETIC INTERACTIONS

Let us take the following interactions :

$$\begin{aligned} &\Sigma^0 \rightarrow \pi+\gamma \\ &\eta \rightarrow \pi^++\pi^-+\gamma \\ &\gamma+p \rightarrow n+\pi^+ \\ &\gamma+p \rightarrow \pi+K^+ \end{aligned} \qquad .. \quad (43)$$

It appears that in these processes, $|\Delta V_3| = 0$. Thus, $V_3$ is conserved in electromagnetic interactions.

## WEAK INTERACTIONS

Let us consider the weak interactions :

$$\Delta \rightarrow p+\pi^-, \quad n+\pi^0$$

$$\Sigma^+ \rightarrow p+\pi^0,$$

$$K \rightarrow 2\pi,\ 3\pi \text{ etc.}$$

$$\Sigma^- \rightarrow n+\pi^-$$

$$\Xi^- \rightarrow \Lambda+\pi^- \tag{44}$$

and so on

In all cases, the change in $V_3$ appears, with reference to (7) and (8), to be

$$|\Delta V_3| = \tfrac{1}{2} \quad .. \tag{45}$$

It appears that $V$-spin is also similar to $I$-spin in respect of weak interactions. This is quite in agreement with (4).

## ACKNOWLEDGEMENTS

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## APPENDIX

From (8), we have for mass,

$$\Sigma_v^0 = <\Sigma_v^0|m|\Sigma_v^0 = \tfrac{1}{4}\Sigma^0+\tfrac{3}{4}\Lambda+\sqrt{3}(\Sigma^0\Lambda)$$

$$(\Sigma_v^0\Lambda_v) = <\Sigma_v^0|m|\Lambda_v> = \frac{\sqrt{3}}{4}\Sigma^0+\frac{\sqrt{3}}{4}\Lambda+(\Sigma^0\Lambda)$$

Now, from (30), we have the identity :

$$n-p+\Sigma^0-\Sigma^-+\alpha(\Sigma^0\Lambda)$$

$$\equiv n-p+\left(\frac{\beta}{4}+\frac{\sqrt{3}}{4}\gamma\right)\Sigma^0+\left(\frac{3\beta}{4}+\frac{\sqrt{3}}{4}\gamma\right)\Lambda-\Sigma^-+\left(\frac{\sqrt{3}}{2}\beta+\gamma\right)(\Sigma^0\Lambda)$$

Hence, we obtain the following equations :

$$\frac{\beta}{4}+\frac{\sqrt{3}}{4}\gamma = 1$$

$$\frac{3\beta}{4} + \frac{\sqrt{3}}{4}\gamma = 0$$

$$\frac{\sqrt{3}}{2}\beta + \gamma = \alpha$$

Solving these equations

$$\alpha = \sqrt{3}, \qquad \beta = -2, \qquad \gamma = 2\sqrt{3}$$

Also, solving in the same manner the identity for the $I$-$U$ parallelogram ($n$, $p$, $\Sigma^+$, $\Sigma^0$), the values obtained are

$$\alpha = \sqrt{3}, \quad \beta = 1, \quad \gamma = \sqrt{3}$$

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